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Short Communication

A note on maximum work from an electric battery model

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Abstract

The maximum work from an electric battery in a finite time interval t_C derived in a recent article in this journal is unreasonable. The reason is discussed in detail and the genuine maximum work is given. This shows that the result derived in the recent article is suitable only in the case $t_C \to \infty$. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

The maximum work from an electric battery in a finite time interval $0 \to t_C$ was derived in a recent article in this journal [1]. It relied on a simple battery model as shown in Fig. 1. The non-dimensionalized maximum work output derived by Bejan and Dan [1] is given by

$$\tilde{W}_{\max} = K[K_1 \cosh(2a\tilde{t}_C) + K_2 \sinh(2a\tilde{t}_C) + K_3 + K_4], \tag{1}$$

but it can be written in a more compact form as

$$\tilde{W}_{\max} = 1 + 2r - 2ar \cosh(a\tilde{t}_C)/\sinh(a\tilde{t}_C), \tag{2}$$

where $K=[2a \sinh^2(a\tilde{t}_C)]^{-1}$, $K_1=a(1+2r)$, $K_2=-r(1+a^2)-1$ (in Ref. [1], $K_2=-r^2(1+a^2)-1$ is a typo), $K_3=2a\tilde{t}_C-a$, $K_4=r[(2a-2a^3)\tilde{t}_C-2a]$, $a^2=1+1/r$ (in Ref. [1], $a=1+1/r$ is a typo), $r=R_m/R_b$, $\tilde{t}_C=t_C/(R_bC)$ and $E=U_C(0)$, i.e., the voltage $U(t)$ of the battery terminals at the initial time $t=0$. The definitions of R_m, R_b, U_m, C, I, I_b and I_m are shown in Fig. 1 and the Nomenclature.

In Ref. [1], the ideas that the work output and the regime of operation of a battery can be optimized are correct and some significant results are obtained. However, Eq. (1) is unreasonable, because $\tilde{W}_{\max}$ can become negative (see Appendix A) when

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Nomenclature

C	capacitance
I	total current
I_b	leakage current
I_m	motor current
R_b	leakage resistance
R_m	motor winding resistance
U_m	voltage between the motor terminals

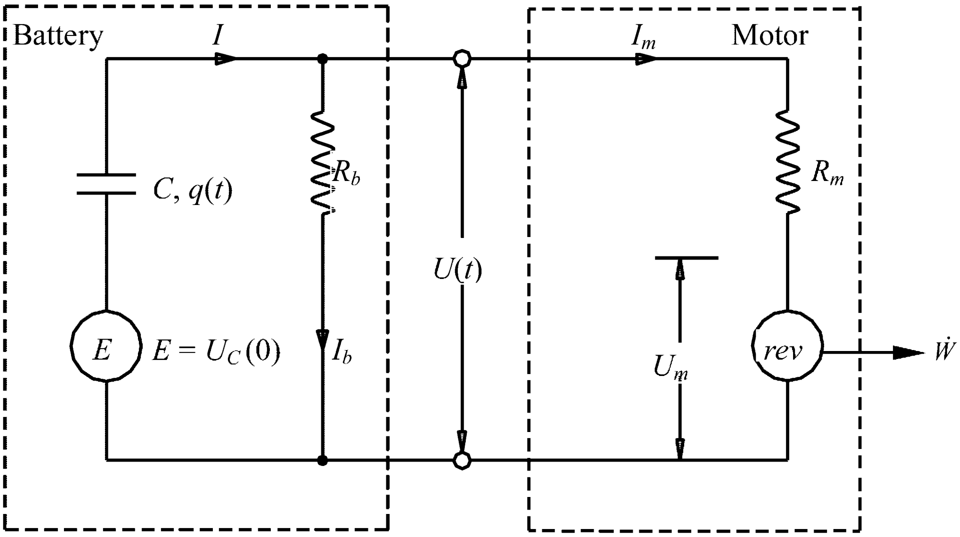

Fig. 1. A simple battery model connected to a DC motor with its winding resistance R_m.

$$t_C < \frac{R_b C}{a} \sinh^{-1}(2ar). \tag{3}$$

Thus, Eq. (1) cannot represent the maximum mechanical work that can be extracted from an electric battery in a finite time interval and it is worthwhile to discuss the problem further.

2. Discussion on the negative maximum work

The maximum mechanical work derived in Ref. [1] can obviously become negative because the instantaneous mechanical power output of the motor (as a load of the battery, see Fig. 1) connected to the battery,

$$\dot{W} = (U - R_m I_m) I_m, \tag{4}$$

is negative when

$$U<R_{\mathrm{m}}I_{\mathrm{m}}. \tag{5}$$

According to the non-dimensionalized optimal regime of time-dependent operation of the battery voltage and the corresponding non-dimensionalized optimal current regime obtained in Ref. [1]:

$$\tilde{U}_{\mathrm{opt}}=\cosh(a\tilde{t})-\frac{\cosh(a\tilde{t}_{\mathrm{C}})}{\sinh(a\tilde{t}_{\mathrm{C}})}\sinh(a\tilde{t}) \tag{6}$$

and

$$\tilde{I}_{\mathrm{m,opt}}=\left[\frac{a\cosh(a\tilde{t}_{\mathrm{C}})}{\sinh(a\tilde{t}_{\mathrm{C}})}-1\right]\cosh(a\tilde{t})+\left[\frac{\cosh(a\tilde{t}_{\mathrm{C}})}{\sinh(a\tilde{t}_{\mathrm{C}})}-a\right]\sinh(a\tilde{t}), \tag{7}$$

one can show that (see Appendix B) when the operating time

$$t>\frac{R_{\mathrm{b}}C}{a}\tanh^{-1}\left[\frac{a\tanh(a\tilde{t}_{\mathrm{C}})-1}{a-\tanh(a\tilde{t}_{\mathrm{C}})}\right]\equiv t_0, \tag{8}$$

the corresponding optimal instantaneous power output $\dot{W}_{\mathrm{max}}$ is negative. Here t_0 may be referred to as the critical operating time. That is, in the time interval $t_0\to t_{\mathrm{C}}$, the power output of the battery is always negative. Eq. (8) clearly shows that as long as t_{C} is finite, t_0 is always less than t_{C} and, thus, a time interval $t_{\mathrm{C}}-t_0$ of negative instantaneous power output will arise. The shorter the operating time interval t_{C}, the more serious the effect of negative instantaneous power output will be. When the absolute value of the negative work produced during the time interval $t_0\to t_{\mathrm{C}}$ is larger than the positive work produced during the time interval $0\to t_0$, the total work output in the time interval $0\to t_{\mathrm{C}}$ is negative. This shows further that the maximum work output derived in Ref. [1] is unreasonable.

The non-dimensionalized total positive work produced during the time interval $0\to t_0$,

$$\tilde{W}_{\mathrm{p}}(\tilde{t}_0)=1+2r-2ar\cosh(a\tilde{t}_{\mathrm{C}})/\sinh(a\tilde{t}_{\mathrm{C}})+r/\sinh^2(a\tilde{t}_{\mathrm{C}}), \tag{9}$$

can be derived from Eqs. (6)–(8). Obviously, $\tilde{W}_{\mathrm{p}}(\tilde{t}_0)$ is larger than $\tilde{W}_{\mathrm{max}}$. Only in the case $t_{\mathrm{C}}\to\infty$ or $r=0$ can one have $\tilde{W}_{\mathrm{p}}(\tilde{t}_0)=\tilde{W}_{\mathrm{max}}$.

Of course, if t_{C} is large enough or r is very small, such that the departure of $\tilde{W}_{\mathrm{max}}$ from $\tilde{W}_{\mathrm{p}}(\tilde{t}_0)$ is negligible, the result in Ref. [1] is approximately suitable. However, in the final analysis, the result in Ref. [1] is unreasonable and cannot represent the genuine maximum mechanical work that can be extracted from a battery in a finite time interval.

3. The genuine maximum work

Now that $\tilde{W}_{\mathrm{p}}(\tilde{t}_0)$ is larger than $\tilde{W}_{\mathrm{max}}$, can $\tilde{W}_{\mathrm{p}}(\tilde{t}_0)$ represent the maximum work that can be extracted from a battery in a finite time interval? The answer is negative; $\tilde{W}_{\mathrm{p}}(\tilde{t})$ is still not the maximum mechanical work. The genuine non-dimensionalized maximum mechanical work that can be extracted from a battery in a finite time interval t_{C} is (see Appendix C)

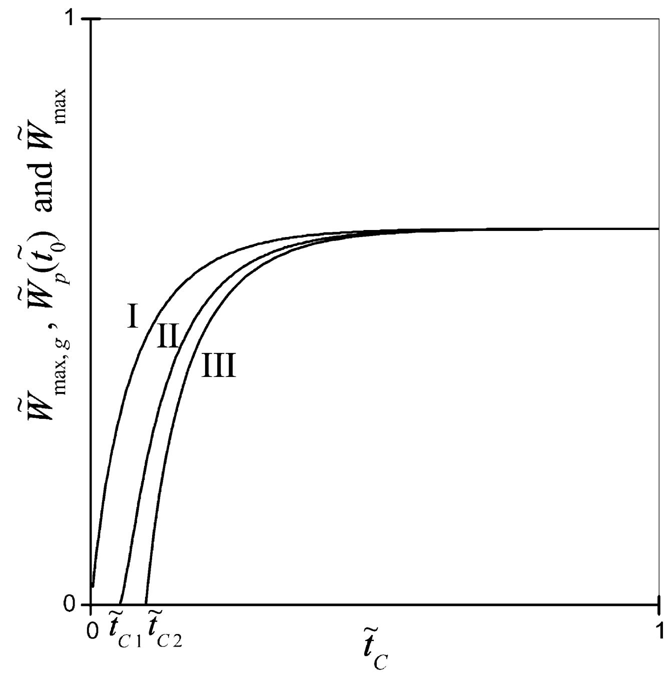

Fig. 2. Curves of $\tilde{W}_{max,g}$ versus $\tilde{t}_C$ (curve I), $\tilde{W}_p(\tilde{t}_0)$ versus $\tilde{t}_C$ (curve II) and $\tilde{W}_{max}$ versus $\tilde{t}_C$ (curve III) for r=0.05, where $\tilde{t}_{C_1}=(1/a)\tanh^{-1}(1/a)$ and $\tilde{t}_{C_2}=(1/a)\sinh^{-1}(2ar)$.

$$\tilde{W}_{max,g}=\frac{1}{1+2r+2ar\cosh(a\tilde{t}_C)/\sinh(a\tilde{t}_C)}. \tag{10}$$

$\tilde{W}_{max,g}>\tilde{W}_p(\tilde{t}_0)>\tilde{W}_{max}$ is shown clearly in Fig. 2. Only in the case $t_C\to\infty$ or r=0 do they tend to the same value. To sum up, $\tilde{W}_{max}$ derived in Ref. [1] cannot represent the maximum mechanical work that can be extracted from an electric battery in a finite time interval.

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Appendix A

From Eq. (3) and $1+r=a^2r$, one has

$$\sinh^2(a\tilde{t}_C)<4a^2r^2=(1+2r)^2-1, \tag{A1}$$

such that there is

$$4a^2r^2\cosh^2(a\tilde{t}_C)/\sinh^2(a\tilde{t}_C)>1+4a^2r^2=(1+2r)^2. \tag{A2}$$

Therefore, $\tilde{W}_{max}$ becomes negative.

Appendix B

From Eq. (8) and $1+r=a^2r$, one has

$$[a\cosh(a\tilde{t}_C)-\sinh(a\tilde{t}_C)]\sinh(a\tilde{t})>[a\sinh(a\tilde{t}_C)-\cosh(a\tilde{t}_C)]\cosh(a\tilde{t}), \tag{B1}$$

such that there is

$$[ar\cosh(a\tilde{t}_C)-(1+r)\sinh(a\tilde{t}_C)]\cosh(a\tilde{t})-[ar\sinh(a\tilde{t}_C)-(1+r)\cosh(a\tilde{t}_C)]\sinh(a\tilde{t})>0. \tag{B2}$$

Therefore, $\tilde{I}_{m,opt}r-\tilde{U}_{opt}>0$ and $\dot{W}_{max}$ is negative.

Appendix C

In order to obtain the genuine maximum work output, we should remove the unsuitable final condition $U_{opt}(t_C)=0$ used in Ref. [1] and suppose $U_{opt}(t_C)=\epsilon E$ (ϵ is less than 1 and should be determined by the maximum work condition). Then, using the same procedure as that used in Ref. [1], one can obtain

$$\tilde{U}_{opt}=\cosh(a\tilde{t})-\frac{\cosh(a\tilde{t}_C)-\epsilon}{\sinh(a\tilde{t}_C)}\sinh(a\tilde{t}), \tag{C1}$$

$$\tilde{I}_{m,opt}=\left[a\frac{\cosh(a\tilde{t}_C)-\epsilon}{\sinh(a\tilde{t}_C)}-1\right]\cosh(a\tilde{t})+\left[\frac{\cosh(a\tilde{t}_C)-\epsilon}{\sinh(a\tilde{t}_C)}-a\right]\sinh(a\tilde{t}) \tag{C2}$$

and the non-dimensionalized maximum work output for a given ϵ:

$$\tilde{W}_{max,\epsilon}=(1+2r)(1-\epsilon^2)-\frac{2ar\cosh(a\tilde{t}_C)}{\sinh(a\tilde{t}_C)}\left[1+\epsilon^2-\frac{2\epsilon}{\cosh(a\tilde{t}_C)}\right]. \tag{C3}$$

Again, using the extremal condition $\partial\tilde{W}_{max,\epsilon}/\partial\epsilon=0$, from Eq. (C3) we can find the genuine maximum work output as shown in Eq. (10).

Reference

[1] Bejan A, Dan N. Maximum work from an electric battery model. Energy 1997;22:97–102.